One-loop mass shift formula for kinks and self-dual vortices

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Abstract

A formula is derived that allows us to compute one-loop mass shifts for kinks and self-dual Abrikosov-Nielsen-Olesen vortices. The procedure is based in canonical quantization and heat kernel/zeta function regularization methods.

1 Introduction

Abrikosov vortex lines [1] were rediscovered by Nielsen and Olesen in the realm of the Abelian Higgs model and were proposed as models for dual strings [2]. In this framework, the interest of studying the quantum nature of these quasi-one-dimensional extended structures was immediately recognized; contrarily to the macroscopic Ginzburg-Landau theory of Type II superconductors, the birth-place of magnetic flux tubes, the Abelian Higgs model is expected to play a rôle in microscopic physics. This issue was first addressed in Section §. 3 of the original Nielsen-Olesen paper; taking the zero thickness limit of the vortex line, the quantization techniques of the old string theory were applied.

In this communication we shall deal with one-loop mass shifts for the topological solitons that generate the thick string structures. The mass of the topological solitons of the (2+1)-dimensional Abelian Higgs model become the string tension of the flux tubes embedded in three dimensions. Thus, from the (3+1)-dimensional perspective, semi-classical string tensions will be considered. In particular we offer as a novelty the derivation of the vortex Casimir energy from the canonical quantization of the planar Abelian Higgs model. With this demonstration, we shall arrive at the starting point chosen in References [3] and [4] to derive a formula for the one-loop vortex mass shifts. The formula involves the coefficients of the heat-kernel expansion associated with the second-order fluctuation operator and affords us a numerical computation of the mass shifts. Before our work, only fermionic fluctuations on vortex backgrounds have been accounted for by Bordag in [5].

The control of the ultra-violet divergences arising in the procedure will be achieved by using heat kernel/zeta function regularization methods. In the absence of detailed knowledge of the spectrum of the differential operator governing second-order fluctuations around vortices, the expansion of the associated heat kernel will be used in a way akin to that developed in the computation of one-loop mass shifts for one-dimensional kinks, see [6], [7], [8]. In fact, a similar technique has been applied before to compute the mass shift for the supersymmetric kink [9], although in this latter case the boundary conditions must respect supersymmetry. In the case of vortices, the only available results besides the work reported here refer to supersymmetric vortices and were achieved by Vassilevich and the Stony Brook/Wien group, [10], [11].

2 High-temperature one-loop kink mass shift formula

We start by very briefly treating the parallel and simpler development for the kink of the $\lambda(\phi)^4_2$-model. With the conventions of [6] the one-loop kink mass shift formula in the $\lambda(\phi)^4_2$ model on a line is formally: $\Delta M_K = \Delta M_K^C + \Delta M_K^R$. The two pieces are:

1. The kink Casimir energy measured with respect to the vacuum Casimir energy -zero point energy renormalization-:
$$\Delta M_K^C = \Delta E(\phi_K) - \Delta E_0 = \frac{\hbar m}{2\sqrt{2}}\left(\mathrm{Tr}K^{\frac{1}{2}} - \mathrm{Tr}K_0^{\frac{1}{2}}\right) \tag{1}$$
$$K = -\frac{d^2}{dx^2} + 4 - \frac{6}{\cosh^2 x} \quad , \quad K_0 = -\frac{d^2}{dx^2} + 4 \quad .$$
K and K_0 are the differential operators governing the second-order fluctuations around the kink and the vacuum respectively.

2. The contribution of the mass renormalization counter-terms to the one-loop kink mass:
$$\Delta M_K^R = -3\frac{\hbar m}{\sqrt{2}} \cdot I(4) \cdot \int dx\, \left(\phi_K^2(x) - \phi_\pm^2\right) \quad , \quad I(4) = \int \frac{d^2k}{(2\pi)^2} \cdot \frac{i}{(k_0^2 - k^2 - 4 + i\varepsilon)} \quad . \tag{2}$$

The ultraviolet divergences are regularized by means of generalized zeta functions:

$$\Delta M_K^C(s) = \frac{\hbar}{2}(2\frac{\mu^2}{m^2})^s \mu\left(\zeta_K(s) - \zeta_{K_0}(s)\right) \quad , \quad \Delta M_K^R(s) = -\frac{6\hbar}{L} \cdot \left(\frac{2\mu^2}{m^2}\right)^{s+\frac{1}{2}} \cdot \frac{\Gamma(s+1)}{\Gamma(s)} \cdot \zeta_{K_0}(s+1) \quad ,$$

where s is a complex parameter; μ a parameter of dimension L^{-1}, and L is a normalization length on the line. From the partition functions, one obtains the generalized zeta functions via Mellin transform:

$$\mathrm{Tr}e^{-\beta K_0} = \frac{mL}{\sqrt{8\pi\beta}} \cdot e^{-4\beta} \quad , \quad \mathrm{Tr}^* e^{-\beta K} = \frac{mL}{\sqrt{8\pi\beta}} \cdot e^{-4\beta} + e^{-3\beta}(1 - \mathrm{Erfc}\sqrt{\beta}) - \mathrm{Erfc}2\sqrt{\beta}$$

$$\zeta_{K_0}(s) = \frac{mL}{\sqrt{8\pi}} \cdot \frac{\Gamma(s-\frac{1}{2})}{2^{2s-1}\Gamma(s)} \quad ; \quad \zeta_K^*(s) = \zeta_{K_0}(s) + \frac{\Gamma(s+\frac{1}{2})}{\sqrt{\pi}\Gamma(s)}\left[\frac{2}{3^{s+\frac{1}{2}}} \cdot {}_2F_1[\frac{1}{2}, s+\frac{1}{2}, \frac{3}{2}, -\frac{1}{3}] - \frac{1}{4^s}\frac{1}{s}\right] \quad ,$$

passing from complementary error functions $\mathrm{Erfc}x$ to hypergeometric Gauss functions ${}_2F_1[a,b,c;d]$. The star means that the zero mode is not accounted for. Because $\Delta M_K^C = \lim_{s\to-\frac{1}{2}} \Delta M_K^C(s)$ and $\Delta M_K^R = \lim_{s\to-\frac{1}{2}} \Delta M_K^R(s)$

$$\Delta M_K^C = -\frac{\hbar m}{2\sqrt{2}\pi}\lim_{\varepsilon\to 0}\left[\frac{3}{\varepsilon} + 3\ln\frac{2\mu^2}{m^2} - \frac{\pi}{\sqrt{3}}\right] \quad , \quad \Delta M_K^R = \frac{\hbar m}{2\sqrt{2}\pi}\lim_{\varepsilon\to 0}\left[\frac{3}{\varepsilon} + 3\ln\frac{2\mu^2}{m^2} - 2(2+1)\right]$$

provides the exact Dashen-Hasslacher-Neveu (DHN) result, see [6] and References quoted therein:

$$\boxed{\Delta M_K = \Delta M_K^C + \Delta M_K^R = \frac{\hbar m}{2\sqrt{6}} - \frac{3\hbar m}{\pi\sqrt{2}}} \quad .$$

Without using the knowledge of the spectrum of K, one can rely on the high-temperature expansion of the partition function:

$$\mathrm{Tr}e^{-\beta K} = \frac{e^{-4\beta}}{\sqrt{4\pi\beta}} \cdot \sum_{n=0}^{\infty} c_n(K)\,\beta^n \quad , \quad c_0(K) = \lim_{L\to\infty}\frac{mL}{\sqrt{2}} \quad , \quad c_n(K) = \frac{2^{n+1}(1+2^{2n-1})}{(2n-1)!!} , \quad n \geq 1$$

to find:

$$\zeta_K(s) = \frac{1}{\Gamma(s)}\left[\frac{1}{\sqrt{4\pi}}\cdot\sum_{n=0}^{\infty} c_n(K)\cdot\frac{\gamma[s+n-\frac{1}{2},4]}{4^{s+n-\frac{1}{2}}} - \int_0^1 d\beta\,\beta^{s-1}\right] + \int_1^{\infty} d\beta\,\beta^{s-1}\mathrm{Tr}^* e^{-\beta K} \qquad .$$

Here, the zero mode has been subtracted and the meromorphic structure of $\zeta_K(s)$ is encoded in the incomplete Gamma functions $\gamma[s+n-\frac{1}{2},4]$. Neglecting the (very small) contribution of the entire function, and cutting the series at a large but finite N_0, the kink Casimir energy becomes:

$$\Delta M_K^C \simeq \frac{\hbar}{2}\cdot\lim_{s\to-\frac{1}{2}}\left(\frac{2\mu^2}{m^2}\right)^s\cdot\mu\cdot\frac{1}{\Gamma(s)}\cdot\left[\frac{1}{\sqrt{4\pi}}\sum_{n=1}^{N_0} c_n(K)\frac{\gamma[s+n-\frac{1}{2},4]}{4^{s+n-\frac{1}{2}}} - \frac{1}{s}\right] \qquad ,$$

i.e. the zero-point vacuum energy renormalization takes care of the term coming from $c_0(K)$. Note that $\zeta_{K_0}(s) \simeq \frac{mL}{\sqrt{8\pi}}\cdot\frac{\gamma[s-\frac{1}{2},4]}{2^{2s-1}\Gamma(s)}$ in the $\beta<1$ regime where the high-T expansion is reliable. The other correction due to the mass renormalization counter-terms can be arranged also into meromorphic and entire parts:

$$\Delta M_K^R = -\frac{\hbar\mu}{2\sqrt{4\pi}}\cdot c_1(K)\cdot\lim_{s\to-\frac{1}{2}}\left(\frac{2\mu^2}{m^2}\right)^{s+\frac{1}{2}}\cdot\frac{1}{4^{s+\frac{1}{2}}\Gamma(s)}\cdot\left[\gamma[s+\frac{1}{2},4]+\Gamma[s+\frac{1}{2},4]\right]$$

The mass renormalization term exactly cancels the $c_1(K)$ contribution. Our minimal subtraction scheme fits with the renormalization prescription set in [12]: for theories with only massive fluctuations, the quantum corrections should vanish in the limit in which all masses go to infinity. This criterion requires precisely the cancelation found. We end with the high-temperature one-loop kink mass shift formula:

$$\boxed{\Delta M_K = -\frac{\hbar m}{4\sqrt{2\pi}}\cdot\left[\frac{1}{\sqrt{4\pi}}\cdot\sum_{n=2}^{N_0} c_n(K)\cdot\frac{\gamma[n-1,4]}{4^{n-1}}+2\right] \quad , \qquad \beta=\frac{\hbar m}{k_B T}<1}$$

Finally, by applying this formula with $N_0 = 11$ we have:

$$\Delta M_K \cong -0.471371\hbar m \qquad ,$$

with an error with respect to the DHN result of: $0.0002580\hbar m$.

3 The planar Abelian Higgs model

In this Section we generalize formulae (1) and (2) to determine the one-loop mass shift of vortices in the Abelian Higgs model. We shall derive the formula that serves as the starting point in papers [3] and [4], thus filling a gap in the issue. Within the conventions stated in these References, we write the action governing the dynamics of the AHM in the form:

$$S = \frac{v}{e}\int d^3x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}(D_\mu\phi)^* D^\mu\phi - \frac{\kappa^2}{8}(\phi^*\phi-1)^2\right] \qquad .$$

A shift of the complex scalar field from the vacuum $\phi(x^\mu) = 1+H(x^\mu)+iG(x^\mu)$ and choice of the Feynman-'t Hooft renormalizable gauge $R(A_\mu, G) = \partial_\mu A^\mu(x^\mu) - G(x^\mu)$ lead us to write the action in terms of Higgs H, Goldstone G, vector boson A_μ and ghost χ fields:

$$\begin{aligned} S + S_{\rm g.f.} + S_{\rm ghost} &= \frac{v}{e}\int d^3x\left[-\frac{1}{2}A_\mu[-g^{\mu\nu}(\Box+1)]A_\nu + \partial_\mu\chi^*\partial^\mu\chi - \chi^*\chi + \frac{1}{2}\partial_\mu G\partial^\mu G - \frac{1}{2}G^2\right.\\ &+ \frac{1}{2}\partial_\mu H\partial^\mu H - \frac{\kappa^2}{2}H^2 - \frac{\kappa^2}{2}H(H^2+G^2) + H(A_\mu A^\mu - \chi^*\chi)\\ &+ \left. A_\mu(\partial^\mu HG - \partial^\mu GH) - \frac{\kappa^2}{8}(H^2+G^2)^2 + \frac{1}{2}(G^2+H^2)A_\mu A^\mu\right] \qquad . \end{aligned}$$

3.1 Vacuum Casimir energy

Canonical quantization promoting the coefficients of the plane wave expansion around the vacuum of the fields to operators provides the free quantum Hamiltonian:

- If $m = ev$,

$$\delta A_\mu(x_0,\vec{x}) = \left(\frac{\hbar^{\frac{1}{2}}}{e^{\frac{1}{2}}v^{\frac{3}{2}}L}\right)\cdot\sum_{\vec{k}}\sum_{\alpha}\frac{1}{\sqrt{2\omega(\vec{k})}}\left[a^*_\alpha(\vec{k})e^\alpha_\mu(k)e^{ikx}+a_\alpha(\vec{k})e^\alpha_\mu(k)e^{-ikx}\right]$$

$$[\hat{a}_\alpha(\vec{k}),\hat{a}^\dagger_\alpha(\vec{q})] = (-1)^{\delta_{\alpha 0}}\delta_{\alpha\beta}\delta_{\vec{k}\vec{q}} \Rightarrow H^{(2)}[\delta\hat{A}_\mu] = \sum_{\vec{k}}\sum_{\alpha}\hbar m\omega(\vec{k})\left((-1)^{\delta_{\alpha 0}}\hat{a}^\dagger_\alpha(\vec{k})\hat{a}_\alpha(\vec{k})+\frac{1}{2})\right) \quad .$$

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$$\delta H(x_0,\vec{x}) = \frac{1}{vL}\sqrt{\frac{\hbar}{ev}}\sum_{\vec{k}}\frac{1}{\sqrt{2\nu(\vec{k})}}\left[a^*(\vec{k})e^{ikx}+a(\vec{k})e^{-ikx}\right] \quad , \quad \nu(\vec{k}) = +\sqrt{\vec{k}\vec{k}+\kappa^2}$$

$$[\hat{a}(\vec{k}),\hat{a}^\dagger(\vec{q})] = \delta_{\vec{k}\vec{q}} \Rightarrow H^{(2)}[\delta\hat{H}] = \hbar m\sum_{\vec{k}}\omega(k)\left(\hat{a}^\dagger(\vec{k})\hat{a}(\vec{k})+\frac{1}{2}\right) \quad .$$

-

$$\delta G(x_0,\vec{x}) = \frac{1}{vL}\sqrt{\frac{\hbar}{ev}}\sum_{\vec{k}}\frac{1}{\sqrt{2\omega(\vec{k})}}\left[b^*(\vec{k})e^{ikx}+b(\vec{k})e^{-ikx}\right] \quad , \quad \omega(\vec{k}) = +\sqrt{\vec{k}\vec{k}+1}$$

$$[\hat{b}(\vec{k}),\hat{b}^\dagger(\vec{q})] = \delta_{\vec{k}\vec{q}} \Rightarrow H^{(2)}[\delta\hat{H}] = \hbar m\sum_{\vec{k}}\omega(k)\left(\hat{b}^\dagger(\vec{k})\hat{b}(\vec{k})+\frac{1}{2}\right) \quad .$$

- Canonical quantization proceeds by anti-commutators for ghost fields

$$\delta\chi(x_0,\vec{x}) = \frac{1}{vL}\sqrt{\frac{\hbar}{ev}}\sum_{\vec{k}}\frac{1}{\sqrt{2\omega(\vec{k})}}\left[c(\vec{k})e^{ikx}+d^*(\vec{k})e^{-ikx}\right]$$

$$\{\hat{c}^\dagger(\vec{k}),\hat{c}(\vec{q})\} = \{\hat{d}^\dagger(\vec{k}),\hat{d}(\vec{q})\} = \delta_{\vec{k}\vec{q}} \Rightarrow H^{(2)}[\delta\hat{\chi}] = \hbar m\sum_{\vec{k}}\omega(k)\left(\hat{c}^\dagger(\vec{k})\hat{c}(\vec{k})+\hat{d}^\dagger(\vec{k})\hat{d}(\vec{k})-1\right) \quad .$$

The vacuum Casimir energy is the sum of four contributions: if $\triangle = \sum_{j=1}^2 \frac{\partial}{\partial x_j}\cdot\frac{\partial}{\partial x_j}$ denotes the Laplacian,

$$\Delta E_0^{(1)} = \sum_{\vec{k}}\sum_{\alpha}\frac{\hbar m}{2}\omega(\vec{k}) = \frac{3\hbar m}{2}\mathrm{Tr}[-\triangle+1]^{\frac{1}{2}} \ , \ \Delta E_0^{(2)} = \sum_{\vec{k}}\frac{\hbar m}{2}\nu(\vec{k}) = \frac{\hbar m}{2}\mathrm{Tr}[-\triangle+\kappa^2]^{\frac{1}{2}}$$

$$\Delta E_0^{(3)} = \sum_{\vec{k}}\frac{\hbar m}{2}\omega(\vec{k}) = \frac{\hbar m}{2}\mathrm{Tr}[-\triangle+1]^{\frac{1}{2}} \quad , \quad \Delta E_0^{(4)} = -\sum_{\vec{k}}\hbar m\omega(\vec{k}) = -\hbar m\mathrm{Tr}[-\triangle+1]^{\frac{1}{2}}$$

come from the vacuum fluctuations of the vector boson, Higgs, Goldstone and ghost fields. Ghost fluctuations, however, cancel the contribution of temporal vector bosons and Golstone particles, and the vacuum Casimir energy in the planar AHM is due only to Higgs particles and transverse massive vector bosons:

$$\Delta E_0 = \sum_{r=1}^{4}\Delta E_0^{(r)} = \hbar m\mathrm{Tr}[-\triangle+1]^{\frac{1}{2}}+\frac{\hbar m}{2}\mathrm{Tr}[-\triangle+\kappa^2]^{\frac{1}{2}} \quad .$$

3.2 Vortex Casimir energy

At the critical point between Type I and Type II superconductivity, $\kappa^2 = 1$, the energy can be arranged in a Bogomolny splitting:

$$E = v^2 \int \frac{d^2x}{2} \left(|D_1\phi \pm iD_2\phi|^2 + [F_{12} \pm \tfrac{1}{2}(\phi^*\phi - 1)]^2\right) + \frac{1}{2}v^2|g| \quad , \quad g = \int d^2x F_{12} = 2\pi l \ , \ l \in \mathbb{Z} \quad .$$

Therefore, the solutions of the first-order equations

$$D_1\phi \pm iD_2\phi = 0 \quad ; \quad F_{12} \pm \frac{1}{2}(\phi^*\phi - 1) = 0$$

are absolute minima of the energy, hence stable, in each topological sector with a classical mass proportional to the magnetic flux. Assuming a purely vorticial vector field plus the spherically symmetric ansatz

$$\begin{array}{ccc} \phi_1(x_1, x_2) = f(r)\mathrm{cos}l\theta & , & \phi_2(x_1, x_2) = f(r)\mathrm{sin}l\theta \\ A_1(x_1, x_2) = -l\dfrac{\alpha(r)}{r}\mathrm{sin}\theta & , & A_2(x_1, x_2) = l\dfrac{\alpha(r)}{r}\mathrm{cos}\theta \end{array} \quad ,$$

$g = -\oint_{r=\infty} dx_i A_i = -l \oint_{r=\infty} \frac{[x_2dx_1 - x_1dx_2]}{r^2} = 2\pi l$, the first-order equations reduce to

$$\frac{1}{r}\frac{d\alpha}{dr}(r) = \mp\frac{1}{2l}(f^2(r) - 1) \quad , \quad \frac{df}{dr}(r) = \pm\frac{l}{r}f(r)[1 - \alpha(r)] \quad ,$$

to be solved together with the boundary conditions $\lim_{r\to\infty} f(r) = 1$, $\lim_{r\to\infty} \alpha(r) = 1$, $f(0) = 0$, $\alpha(0) = 0$ required by energy finiteness plus regularity at the origin (center of the vortex). A partly numerical, partly analytical procedure provides the field profiles $f(r)$, $\alpha(r)$ as well as the magnetic field and the energy density

$$B(r) = \frac{l}{2r}\frac{d\alpha}{dr} \quad , \quad \varepsilon(r) = \frac{1}{4}(1 - f^2(r))^2 + \frac{l^2}{r^2}(1 - \alpha(r))^2 f^2(r) \quad .$$

plotted in Figure 1 for $l = 1, 2, 3, 4$.

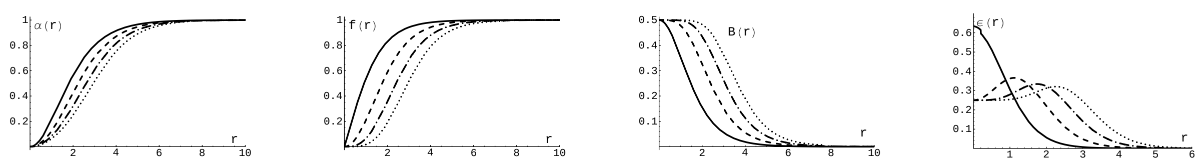

Figure 1. *Plots of the field profiles $\alpha(r)$ (a) and $f(r)$ (b); the magnetic field $B(r)$ (c), and the energy density $\varepsilon(r)$ for self-dual vortices with $l = 1$ (solid line), $l = 2$ (dashed line), $l = 3$ (broken-dashed line) and $l = 4$ (dotted line).*

Consider small fluctuations around vortices $\phi(x_0, \vec{x}) = s(\vec{x}) + \delta s(x_0, \vec{x})$, $A_k(x_0, \vec{x}) = V_k(\vec{x}) + \delta a_k(x_0, \vec{x})$, where by $s(\vec{x})$ and $V_k(\vec{x})$ we respectively denote the scalar and vector field of the vortex solutions. Working in the Weyl/background gauge

$$A_0(x_0, \vec{x}) = 0 \quad , \quad \partial_k \delta a_k(x_0, \vec{x}) + s_2(\vec{x})\delta s_1(x_0, \vec{x}) - s_1(\vec{x})\delta s_2(x_0, \vec{x}) = 0 \quad ,$$

the classical energy up to $\mathcal{O}(\delta^2)$ order is:

$$H^{(2)} + H^{(2)}_{\text{g.f.}} + H^{(2)}_{\text{ghost}} = \frac{v^2}{2} \int d^2x \left\{ \frac{\partial \delta\xi^T}{\partial x_0}\frac{\partial \delta\xi}{\partial x_0} + \delta\xi^T(x_0, \vec{x})K\delta\xi(x_0, \vec{x}) + \delta\chi^*(\vec{x})\left(-\triangle + |s(\vec{x})|^2\right)\delta\chi(\vec{x}) \right\} \quad ,$$

where

$$\delta\xi(x_0, \vec{x}) = \begin{pmatrix} \delta a_1(x_0, \vec{x}) \\ \delta a_2(x_0, \vec{x}) \\ \delta s_1(x_0, \vec{x}) \\ \delta s_2(x_0, \vec{x}) \end{pmatrix} \quad , \quad \nabla_j s_a = \partial_j s_a + \varepsilon_{ab}V_j s_b \quad ,$$

and

$$K = \begin{pmatrix} -\triangle + |s|^2 & 0 & -2\nabla_1 s_2 & 2\nabla_1 s_1 \\ 0 & -\triangle + |s|^2 & -2\nabla_2 s_2 & 2\nabla_2 s_1 \\ -2\nabla_1 s_2 & -2\nabla_2 s_2 & -\triangle + \frac{1}{2}(3|s|^2 + 2V_k V_k - 1) & -2V_k \partial_k \\ 2\nabla_1 s_1 & 2\nabla_2 s_1 & 2V_k \partial_k & -\triangle + \frac{1}{2}(3|s|^2 + 2V_k V_k - 1) \end{pmatrix} .$$

The general solutions of the linearized field equations

$$\frac{\partial^2 \delta\xi_A}{\partial x_0^2}(x_0, \vec{x}) + \sum_{B=1}^{4} K_{AB} \cdot \delta\xi_B(x_0, \vec{x}) = 0 \qquad , \qquad K^G \delta\chi(\vec{x}) = \left(-\triangle + |s(\vec{x})|^2\right) \delta\chi(\vec{x}) = 0$$

are the eigenfunction expansions

$$\delta\xi'_A(x_0, \vec{x}) = \frac{1}{vL}\sqrt{\frac{\hbar}{ev}} \cdot \sum_{\vec{k}} \sum_{I=1}^{4} \frac{1}{\sqrt{2\varepsilon(\vec{k})}} \left[a_I^*(\vec{k}) e^{i\varepsilon(\vec{k})x_0} u_A^{(I)*}(\vec{x};\vec{k}) + a_I(\vec{k}) e^{-i\varepsilon(\vec{k})x_0} u_A^{(I)}(\vec{x};\vec{k})\right]$$

$$\delta\chi'(x_0, \vec{x}) = \frac{1}{vL}\sqrt{\frac{\hbar}{ev}} \cdot \sum_{\vec{k}} \frac{1}{\sqrt{2\varepsilon^G(\vec{k})}} \left[c(\vec{k}) u^*(\vec{x};\vec{k}) + d^*(\vec{k}) u(\vec{x};\vec{k})\right] \qquad ,$$

where $A = 1, 2, 3, 4$ and by $u^{(I)}(k)$, $u(k)$ the non-zero eigenfunctions of K and K^G are denoted respectively: $K u^{(I)}(\vec{x}) = \varepsilon(\vec{k}) u^{(I)}(\vec{x})$, $K^G u(\vec{x}) = \varepsilon^G(\vec{k}) u(\vec{x})$. Canonical quantization

$$[\hat{a}_I(\vec{k}), \hat{a}_J^\dagger(\vec{q})] = \delta_{IJ}\delta_{\vec{k}\vec{q}} \quad , \quad \{\hat{c}(\vec{k}), \hat{c}^\dagger(\vec{q})\} = \delta_{\vec{k}\vec{q}} \quad , \quad \{\hat{d}(\vec{k}), \hat{d}^\dagger(\vec{q})\} = \delta_{\vec{k}\vec{q}}$$

leads to the quantum free Hamiltonian

$$\hat{H}^{(2)} + \hat{H}^{(2)}_{\text{g.f.}} + \hat{H}^{(2)}_{\text{Ghost}} = \hbar m \cdot \sum_{\vec{k}} \left[\sum_{I=1}^{4} \varepsilon(\vec{k}) \left(\hat{a}_I^\dagger(\vec{k}) \hat{a}_I(\vec{k}) + \frac{1}{2}\right) + \frac{1}{2}\varepsilon^G(\vec{k}) \left(\hat{c}^\dagger(\vec{k})\hat{c}(\vec{k}) + \hat{d}^\dagger(\vec{k})\hat{d}(\vec{k}) - 1\right)\right] \quad ,$$

and the vortex Casimir energy reads:

$$\triangle E_V = \frac{\hbar m}{2} \text{STr}^* \, K^{\frac{1}{2}} = \frac{\hbar m}{2} \text{Tr}^* \, K^{\frac{1}{2}} - \frac{\hbar m}{2} \text{Tr}^* \, (K^{\text{G}})^{\frac{1}{2}} \qquad .$$

Note that the ghost fields are static in this combined Weyl-background gauge and their vacuum energy is one-half with respect to the time-dependent case. Only the Goldstone fluctuations around the vortices must be subtracted. The zero-point vacuum energy renormalization provides an analogous formula to (1) for self-dual ($\kappa^2 = 1$) vortices

$$\triangle M_V^C = \triangle E_V - \triangle E_0 = \frac{\hbar m}{2} \left[\text{STr}^* \, K^{\frac{1}{2}} - \text{STr} \, K_0^{\frac{1}{2}}\right] \tag{3}$$

3.3 Mass renormalization energy

Adding the counter-terms

$$\mathcal{L}^S_{c.t.} = 2\hbar I(1) \left[|\phi|^2 - 1\right] \quad , \quad \mathcal{L}^A_{c.t.} = -\hbar I(1) A_\mu A^\mu \quad , \quad I(1) = \int \frac{d^3k}{(2\pi)^3} \cdot \frac{i}{k^2 - 1 + i\varepsilon}$$

to the Lagrangian, the divergences arising in the one-loop Higgs, Goldstone and vector boson self-energy graphs as well as the Higgs tadpole are exactly canceled. These terms add the following contribution to the one-loop vortex mass shift:

$$\triangle M_V^R = \hbar m I(1) \int d^2x \, \left[2(1 - |s(\vec{x})|^2) - V_k(\vec{x}) V_k(\vec{x})\right] \qquad , \tag{4}$$

and, formally, $\triangle M_V = \triangle M_V^C + \triangle M_V^R$.

4 High-temperature one-loop vortex mass shift formula

As in the kink case, from the high-temperature expansion of the heat kernels

$$\mathrm{Tr}e^{-\beta K} = \frac{e^{-\beta}}{4\pi\beta}\cdot\sum_{n=0}^{\infty}\sum_{A=1}^{4}\beta^n [c_n]_{AA}(K) \qquad , \qquad \mathrm{Tr}e^{-\beta K^{\mathrm{G}}} = \frac{e^{-\beta}}{4\pi\beta}\cdot\sum_{n=0}^{\infty}\beta^n c_n(K^{\mathrm{G}})$$

the vortex generalized zeta functions can be written in the form:

$$\zeta_K(s) = \sum_{n=0}^{\infty}\sum_{A=1}^{4}[c_n]_{AA}(K)\cdot\frac{\gamma[s+n-1,1]}{4\pi\Gamma(s)} + \frac{1}{\Gamma(s)}\int_1^{\infty}\mathrm{Tr}^* e^{-\beta K}\, d\beta$$

$$\zeta_{K^G}(s) = \sum_{n=0}^{\infty}c_n(K^G)\cdot\frac{\gamma[s+n-1,1]}{4\pi\Gamma(s)} + \frac{1}{\Gamma(s)}\int_1^{\infty}d\beta\,\mathrm{Tr}^* e^{-\beta K^G} \qquad .$$

Neglecting the entire part and setting a large but finite N_0 the vortex Casimir energy is regularized as

$$\Delta M_V^C(s) = \frac{\hbar\mu}{2}\left(\frac{\mu^2}{m^2}\right)^s\left\{-\frac{2l}{\Gamma(s)}\int_0^1 d\beta\beta^{s-1} + \sum_{n=1}^{N_0}\left[\sum_{A=1}^{4}[c_n]_{AA}(K) - c_n(K^G)\right]\cdot\frac{\gamma[s+n-1,1]}{\Gamma(s)}\right\} \qquad ,$$

where the $2l$ zero modes have been subtracted: the zero-point vacuum renormalization amounts to throwing away the contribution of the $c_0(K)$ and $c_0(K^G)$ coefficients. Also, ΔM_V^R is regularized in a similar way

$$\Delta M_V^R(s) = \frac{\hbar}{2\mu L^2}\left(\frac{\mu^2}{m^2}\right)^s\zeta_{K_0^{\mathrm{G}}}(s)\Sigma(s(\vec{x}),V_k(\vec{x}))\,;\;\Sigma(s(\vec{x}),V_k(\vec{x})) = \int d^2x\,[2(1-|s(\vec{x})|^2) - V_k(\vec{x})V_k(\vec{x})] \qquad .$$

The physical limits $s=-\frac{1}{2}$ for ΔM_V^C and $s=\frac{1}{2}$ for ΔM_V^R are regular points of the zeta functions (contrarily to the kink case). But, as in the kink case, the contribution of the first coefficient of the asymptotic expansion exactly kills the contribution of the mass renormalization counter-terms. The explanation of this fact proceeds along the same lines as in the kink case.

$$\Delta M_V^{(1)C}(-1/2) = -\frac{\hbar m}{8\pi}\Sigma(s,V_k)\cdot\frac{\gamma[-1/2,1]}{\Gamma(1/2)} \qquad , \qquad \Delta M_V^R(1/2) = \frac{\hbar m}{8\pi}\cdot\Sigma(s,V_k)\cdot\frac{\gamma[-1/2,1]}{\Gamma(1/2)}$$

and we finally obtain the high-temperature one-loop vortex mass shift formula:

$$\boxed{\Delta M_V = -\frac{\hbar m}{2}\left[\frac{1}{8\pi\sqrt{\pi}}\cdot\sum_{n=2}^{N_0}\left[\sum_{A=1}^{4}[c_n]_{AA}(K) - c_n(K^{\mathrm{G}})\right]\cdot\gamma[n-\tfrac{3}{2},1] + \frac{2l}{\sqrt{\pi}}\right] \quad , \quad \beta = \frac{\hbar m}{k_B T} < 1} \qquad .$$

Numerical integration of the Seeley densities allows us to compute the heat kernel coefficients. We thus find, by setting $N_0 = 6$, the following numerical results for one-loop mass shifts of superimposed vortices with low magnetic fluxes:

$$M_V^{l=1} = m\left(\frac{\pi v}{e} - 1.09427\hbar\right) + o(\hbar^2) \qquad , \qquad M_V^{l=2} = 2m\left(\frac{\pi v}{e} - 1.08106\hbar\right) + o(\hbar^2)$$

$$M_V^{l=3} = 3m\left(\frac{\pi v}{e} - 1.06230\hbar\right) + o(\hbar^2) \qquad , \qquad M_V^{l=4} = 4m\left(\frac{\pi v}{e} - 1.04651\hbar\right) + o(\hbar^2).$$

5 Summary and outlook

We have offered a parallel exposition of the derivation of formulae giving the semi-classical masses of kinks and self-dual vortices starting from canonical quantization and proceeding through heat kernel expansions/generalized zeta functions methods. The treatment of this problem for these respectively one-dimensional and two-dimensional solitons has thus been unified. It seems compelling to apply this method to compute the semi-classical mass of one or other form of Chern-Simons-Higgs vortices [13].

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